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Dark Matter Constraints from a Cosmic Index of Refraction

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Abstract

The dark-matter candidates of particle physics invariably possess electromagnetic interactions, if only via quantum fluctuations. Taken en masse, dark matter can thus engender an index of refraction which deviates from its vacuum value. Its presence is signaled through frequency-dependent effects: the real part yields dispersive effects in propagation, and the imaginary part yields such in attenuation. We discuss theoretical constraints on the expansion of the index of refraction with frequency, the physical interpretation of the terms, and the particular observations needed to isolate its coefficients. This, with the advent of new opportunities to view gamma-ray bursts at cosmological distance scales, gives us a new probe of dark matter. As a first application we use the time delay determined from radio afterglow observations of gamma-ray bursts to limit the charge-to-mass ratio of dark matter to $|\varepsilon|/M < 1.8 \times 10^{-5}\ \mathrm{eV}^{-1}$ at 95% CL.

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A concordance of astronomical observations from different cosmological epochs speaks to a Universe with dark matter — and dark energy [1–3]. Assuming that the observed effects attributed to dark matter are not due to some canny modification of gravity [4], we wish to discover its nature. Some twenty-three percent of the Universe's energy budget is in dark matter [1–3], yet, despite its abundance, little is known of its properties. A number of methods have been proposed for either the direct or indirect detection of dark matter; such studies typically rely, for direct searches [5, 6], on dark-matter–nucleus scattering, and, for indirect searches [6], on two-body dark-matter annihilation to Standard Model (SM) particles. Regardless, the constraints derive from the aftermath of particular two-body interactions. In this Letter we consider a different tack, suggested by the possibility that dark matter consists of sufficiently low mass particles, be they, e.g., warm thermal relics or axion-like particles, that its number density greatly exceeds that of ordinary matter. We thus consider dark matter as *matter*. We shall use probes of dark matter properties en masse to infer constraints on its particulate nature.

We probe the bulk properties of dark matter by studying the modification of the properties of light upon passage through it. In principle, one can study either polarization [7] or propagation effects. We shall focus exclusively on the latter, so that matter effects are signaled by dispersive effects in the speed or attenuation of light. We study such effects by introducing an index of refraction $n(\omega, z)$, where ω is the angular frequency of the light and z is the redshift at which the matter is located. The quantity $n(\omega, z)$ is controlled at the quantum level by the forward Compton amplitude $\mathcal{M}(k, p \to k, p)$ for light–dark-matter scattering, $\gamma(k) + \chi(p) \to \gamma(k') + \chi(p')$. We emphasize that a dark-matter particle need not have an electric charge to scatter a photon; it need only couple to electromagnetically charged particles to which the photons couple. The Compton amplitude can be related by crossing symmetry to the amplitude for dark-matter annihilation into two photons, namely $\chi(p) + \bar{\chi}(-p') \to \gamma(-k) + \gamma(k')$. Thus any dark-matter model which gives rise to an indirect detection signal via $\chi\bar{\chi} \to \gamma\gamma$ [8] can also drive the index of refraction of light propagating in the dark-matter medium from unity. The forward scattering of photons on dark matter results in a macroscopic refractive index of the cosmos. Its real part is associated with the speed of propagation, whereas its imaginary part is associated with attenuation. If n is dispersive, so that, in particular, its real part is frequency-dependent, it may be possible to observe a frequency-dependent time lag between simultaneously emitted pulses from bright, distant astrophysical sources.

The current limits on the non-observation of frequency-dependent effects in the speed of light are severe. The best limits from terrestrial experiments control the variation in the speed of light c with frequency to $|\delta c|/c \lesssim 1 \times 10^{-8}$ [9]; better tests require the use of astrophysical sources. For example, the arrival time difference of pulses from the Crab nebula bound $|\delta c|/c \lesssim 5 \times 10^{-17}$ [10, 11]; better limits still are possible from the study of gamma-ray bursts (GRBs) [11, 12]. GRBs are bright, violent bursts of high energy photons lasting on the order of fractions to hundreds of seconds, making them an apt source for study; indeed, their brightness makes observation at high redshift possible.

Despite the disparate nature of dark-matter models [6, 13], it is still possible to make model-independent statements about the refractive index of a dark-matter medium. However, we do need to assume that the photon energy is small compared to the energy threshold required to materialize the electromagnetically charged particles to which the dark matter can couple. In models of electroweak-symmetry breaking which address the hierarchy problem, a dark-matter candidate can emerge as a by-product. In such models the inelastic threshold $\omega_{\rm th}$ is commensurate with the weak scale, or crudely with energies in excess of $\mathcal{O}(200\,{\rm GeV})$, as the new particles are produced in pairs. If the photon energy ω satisfies the condition $\omega \ll \omega_{\rm th}$ we can apply the techniques of low-energy physics to the analysis of the forward Compton amplitude. Under an assumption of Lorentz invariance and other symmetries, we can expand the forward Compton amplitude in powers of ω and give a physical interpretation to the coefficients of the first few terms as $\omega \to 0$ [14, 15]. In particular, the term in $\mathcal{O}(\omega^0)$ is controlled by the dark matter particles' charge and mass, weighted by preponderance, irrespective of all other considerations save our assumption of Lorentz invariance [16]. Much of the energy range of the Fermi (GLAST) Telescope, as well as some of that of VERITAS, satisfies the condition $\omega \ll \omega_{\rm th}$ if dark matter is linked to new weak-scale physics. Were dark-matter particles to possess a non-zero electric charge, no matter how small, and have a particle mass greater than that of the electron, the inelastic threshold, rather, would be no larger than that for e^+e^- production.

Observational limits on frequency-dependent variations in the speed of light have long been employed as limits on Lorentz violation [11, 17]. More recently, searches for frequency-dependent time lags from GRB data have also been used to set limits on Lorentz violation and, concomitantly, to place constraints on quantum gravity models [12, 18, 19]. Generally, however, one cannot infer Lorentz violation from a transmission experiment, as Lorentz violation can be confused with matter effects [17]. Here, however, the z dependence of the two scenarios should be different, and space-time foam models [12, 18] can yield energy-dependent effects which do not appear in the analysis of the transmission of light through dark matter.

In what follows, we first construct the explicit connection between the index of refraction and the forward Compton amplitude, and we determine the allowed form of the latter in the limit as $\omega \to 0$, under an assumption of Lorentz invariance and of other symmetries. We discuss the physical interpretation of the first few terms and the observational strategies needed to isolate their coefficients. We then proceed to determine a limit on the coefficient of the leading term as $\omega \to 0$ from fits to the existing gamma-ray burst data; this goes to limiting the electric charge to mass ratio of dark matter. Here we assume a single constituent for simplicity. We discuss the possible limitations of our framework and conclude with a summary and outlook.

The relationship between the index of refraction $n(\omega)$ and the forward scattering amplitude $f_\omega(0)$ for light of angular frequency ω is given by [20]

$$n(\omega) = 1 + \frac{2\pi N}{\omega^2} f_\omega(0)\,, \tag{1}$$

where N denotes the number density of scatterers, and we work in units of $c = \hbar = 1$ here and henceforth. The relationship can be derived by considering the transmission of a wave through a thin slab of material at rest [21], so that Eq. (1) represents the leading deviation of $n(\omega)$ from unity. We wish to relate $f_\omega(0)$ to the matrix element $\mathcal{M}$ of quantum field theory, so that we can connect to particle physics models of dark matter. We do this by computing the differential cross section for forward Compton scattering from a point-like Dirac fermion of charge e and mass M at rest and noting $d\sigma/d\Omega = |f_\omega(0)|^2$. Using the conventions of Ref. [22], we determine $f_\omega(0) = \mathcal{M}_r(k,p \to k,p)/8\pi M$, where the overall phase is fixed by demanding that $\mathcal{M}_r(k,p \to k,p)$ obey the optical theorem. We thus have

$$n(\omega) = 1 + \frac{\rho}{4M^2\omega^2} \mathcal{M}_r(k,p \to k,p)\,, \tag{2}$$

with $\rho = MN$ the mass density of the scatterers. Note that $\mathcal{M}_r(k,p \to k,p)$ is evaluated in the dark-matter rest frame, so that $p = (M, \mathbf{0})$ and $k = (\omega, \omega\hat{\mathbf{n}})$. Under the assumptions of causality or, more strictly, of Lorentz invariance, as well as of charge-conjugation, parity, and time-reversal symmetry in the photon–dark-matter interaction, we have [14]

$$\mathcal{M}_r(k,p \to k,p) = f_1(\omega)\boldsymbol{\epsilon}'^* \cdot \boldsymbol{\epsilon} + i f_2(\omega)\mathcal{S} \cdot \boldsymbol{\epsilon}'^* \times \boldsymbol{\epsilon}\,, \tag{3}$$

where $\mathcal{S}$ is the spin operator associated with the dark-matter particle and $\boldsymbol{\epsilon}$ ($\boldsymbol{\epsilon}'$) is the polarization vector associated with the photon in its initial (final) state. The photon is transverse, so that $\boldsymbol{\epsilon} \cdot \hat{\mathbf{n}} = \boldsymbol{\epsilon}' \cdot \hat{\mathbf{n}} = 0$. The functions $f_1(\omega)$ and $f_2(\omega)$ are even and odd, respectively, under $\omega \to -\omega$ [14, 15]. For $\omega < \omega_{\rm th}$, $f_1(\omega)$ and $f_2(\omega)$ are real. Moreover, $f_i(\omega)$ with $i \in 1, 2$ as $\omega \to 0$ are known *exactly*; no additional assumptions need be made — it does not even matter if the dark-matter particle is composite. We have $\mathrm{Re} f_1(0) = -2\varepsilon^2 e^2$ [16, 22], whereas $\mathrm{Re} f_2(\omega) \sim -2\omega M(\mu/S - \varepsilon e/M)^2$ as $\omega \to 0$ [23], where μ is the total magnetic moment and S is the particle's spin. The amplitude $\mathcal{M}_r(k,p \to k,p)$ is implicitly a 2×2 matrix in the photon polarization, and its diagonal matrix elements describe dispersion in propagation and attenuation. The $f_2(\omega)$ term describes changes in polarization with propagation exclusively, so that we need not consider it further. Under the additional assumptions of analyticity and unitarity, dispersion relations emerge for $f_1(\omega)$ and $f_2(\omega)$. For $f_1(\omega)$ [14, 15],

$$\mathrm{Re} f_1(\omega) - \mathrm{Re} f_1(0) = \frac{4M\omega^2}{\pi} \int_0^\infty \mathrm{d}\omega' \frac{\sigma(\omega')}{\omega'^2 - \omega^2}\,, \tag{4}$$

where we use the optical theorem to replace $\mathrm{Im} f_1(\omega)$ with the unpolarized cross section σ. The integral commences at the inelastic threshold $\omega_{\rm th}$, so that it is well-posed at $\omega' = 0$. Expanding Eq. (4) for $\omega \ll \omega_{\rm th}$ yields a series in positive powers of ω^2; moreover, the coefficient of every term of $\mathcal{O}(\omega^2)$ and higher is positive definite. Thus we see that a term in $n(\omega)$ which is linear in ω, as found in some quantum gravity models [12, 18], does not appear as a modification of the phase velocity from matter effects if $\omega < \omega_{\rm th}$. Parametrizing the forward Compton amplitude as $\mathcal{M}_r = \sum_{j=0,\mathrm{even}} A_j \omega^j$, with A_j real, we thus have

$$n(\omega) = 1 + \frac{\rho}{4M^2\omega^2}\left(A_0 + A_2\omega^2 + \dots\right)\,, \tag{5}$$

where $A_0 = \mathrm{Re} f_1(0)$. The terms in $\mathcal{O}(\omega^2)$ and higher are associated with the polarizabilities of the dark-matter candidate. We now turn to the manner in which we can realize observational constraints on A_i.

The index of refraction fixes the phase velocity v_p in the medium; it is $v_p = \omega/k = 1/\tilde{n}$ with $\tilde{n} \equiv \mathrm{Re}\, n$. Observable dispersive effects in light propagation are controlled, rather, by the group velocity v_g, for which $v_g = \mathrm{d}\omega/\mathrm{d}k = (\tilde{n} + \omega(\mathrm{d}\tilde{n}/\mathrm{d}\omega))^{-1}$. Thus the light emitted from a source at a particular time at a distance l away possesses a frequency-dependent arrival time $t(\omega)$, namely $t(\omega) = l(\tilde{n} + \omega \mathrm{d}\tilde{n}/\mathrm{d}\omega)$, or

$$t(\omega) = l\left(1 + \frac{\rho}{4M^2}\left(\frac{-A_0}{\omega^2} + A_2 + 3A_4\omega^2 + \mathcal{O}(\omega^4)\right)\right)\,. \tag{6}$$

For sources at cosmological distances, we must account for the impact of an expanding universe on the arrival time [19]. As we look back to a light source at redshift z, we note that the dark-matter density accrues a scale factor of $(1+z)^3$, whereas the photon energy is blue shifted by a factor of $1+z$ relative to its present-day value ω_0 [19]. Thus we have the arrival time $t(\omega_0, z)$ for light of observed angular frequency ω_0 from a source with red shift z:

$$t(\omega_0, z) = \int_0^z \frac{\mathrm{d}z'}{H(z')} \left(1 + \frac{\rho_0 (1+z')^3}{4M^2} \left(\frac{-A_0}{((1+z')\omega_0)^2} + A_2 + 3A_4 (1+z')^2 \omega_0^2 + \mathcal{O}(\omega_0^4)\right)\right) \tag{7}$$

with the Hubble rate $H(z') = H_0 \sqrt{(1+z')^3 \Omega_M + \Omega_\Lambda}$. We employ the cosmological parameters determined through the combined analysis of WMAP five-year data in the ΛCDM model with distance measurements from Type Ia supernovae (SN) and with baryon acoustic oscillation information from the distribution of galaxies [3]. Thus the Hubble constant today is $H_0 = 70.5 \pm 1.3\,\mathrm{km\ s^{-1} Mpc^{-1}}$, whereas the fraction of the energy density in matter relative to the critical density today is $\Omega_M = 0.274 \pm 0.015$ and the corresponding fraction of the energy density in the cosmological constant Λ is $\Omega_\Lambda = 0.726 \pm 0.015$ [3]. Various strategies must be employed to isolate the coefficients A_i in Eq. (7). To start we note that the A_2 term incurs no frequency-dependent shift in the speed of light, so that its effects are unobservable with our method. One can only infer its presence if one possesses a distance measure independent of z, much as in the manner one infers a nonzero cosmological constant from SN data. Interestingly, as $A_2 > 0$ it has the same phenomenological effect as a nonzero cosmological constant; the longer arrival time leads to an inferred larger distance scale. Cosmologically, though, its effect is very different as it scales with the dark-matter density; it acts as grey dust. The remaining terms can be constrained by dispersive effects. The term in ω_0^{-2} is best constrained through the arrival time difference of a gamma-ray pulse from a GRB and its radio afterglow. The terms in positive powers of ω_0 are best constrained through arrival time differences of optical or gamma-ray pulses of varying energy.

A nonzero A_0 term would connote the existence of dark matter with a nonzero electric charge. It has long been known that dark-matter cannot have unit electric charge [24], but the possibility of a small fractional electric charge, a “millicharge,” has not been ruled out [25–27]. Millicharged particles arise naturally in models with an additional $U(1)$ gauge group mediated by a new massless boson, a paraphoton [28]; a kinetic mixing term allows for interaction between the two massless bosons. Thus even if the dark matter were to couple directly to the paraphoton exclusively, it would effectively couple to the photon as well by dint of photon-paraphoton mixing. If the mixing angle is small, the photon sees a dark matter particle with a millicharge.

As an example, we now turn to the determination of A_0 from GRB data. From publicly available data [29], we consider all GRBs with known redshift in which a radio afterglow is also detected. We determine the time lag between the initial detection of the GRB at some energy and the detection of the radio afterglow, so that our observable is $\tau = t(\omega_0^{\mathrm{low}}, z) - t(\omega_0^{\mathrm{high}}, z)$. If we first observe the GRB at keV energies and compare with the observed arrival time in radio frequencies, then we expect the terms in positive powers of ω_0 in τ to be negligible. We can also neglect the term in $1/(\omega_0^{\mathrm{high}})^2$, as the factor $(\omega_0^{\mathrm{low}}/\omega_0^{\mathrm{high}})^2$ represents less than an $\mathcal{O}(10^{-12})$ correction to unity; we let $\omega \equiv \omega_0^{\mathrm{low}}$ henceforth. In order to assess reliable limits on A_i from the study of the dispersion in arrival times, we must separate propagation effects from intrinsic source effects. Statistically, we expect time delays intrinsic to the source to be independent of z, and the time delay from propagation to depend on z and ω_0 precisely as indicated in Eq. (7). Thus the two behaviors are distinct. Such notions

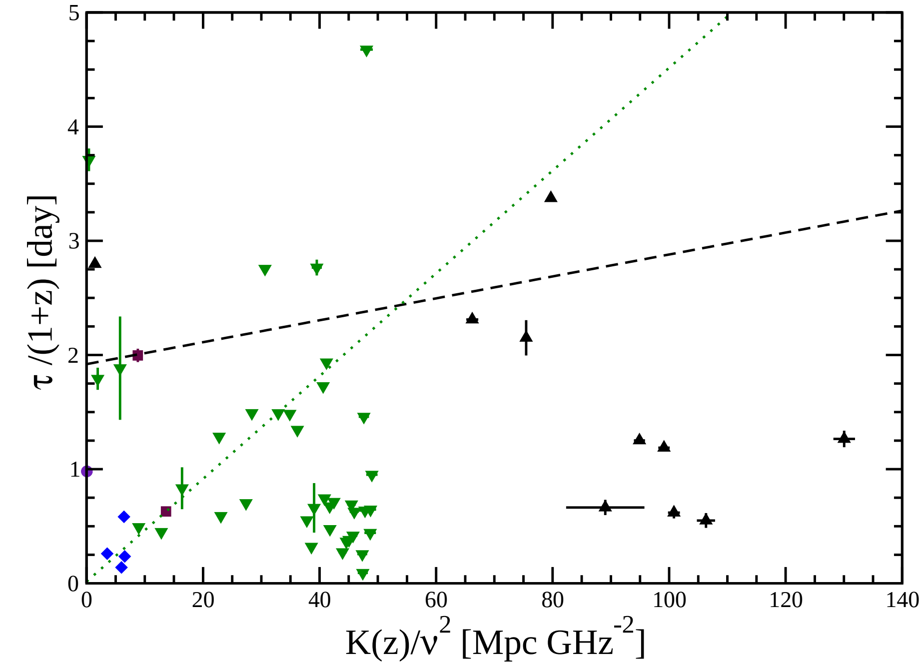

FIG. 1: The time lag τ determined from the observation of a GRB and its radio afterglow plotted as a function of $K(z)/\nu^2$ [29], with $\nu = \omega/2\pi$. The points correspond to frequency windows of $4.8 - 5$ GHz (▲, black), $8.4 - 8.5$ GHz (▼, green), $15 - 16$ GHz (■, maroon), $22 - 22.5$ GHz (♦, blue), and 250 GHz (•, violet). The fits of Eq. (8) to the data corresponding to the central value of $\delta(\nu)$ and the maximum value of the slope $\tilde{A}_0$ at 95% CL are shown as well. Statistical scale factors, as described in text, are needed for fits of good quality but are not shown explicitly. The dashed line results from the fit to the black data, whereas the dotted line results from the fit to the green data. For clarity of presentation we display time lags in the GRB rest frame of less than 5 days only.

have been previously employed in searches for Lorentz invariance violation [18]. Thus we fit

$$\frac{\tau}{1+z} = \tilde{A}_0 \frac{K(z)}{\nu^2} + \delta(\nu)\,, \tag{8}$$

where we introduce the frequency $\nu \equiv \omega/2\pi$ and $K(z) \equiv (1+z)^{-1} \int_0^z \mathrm{d}z\,(1+z')H(z')^{-1}$ and note $4\pi^2\tilde{A}_0 = -A_0\rho_0/4M^2 = 2\pi\alpha\varepsilon^2\rho_0/M^2$ in the Heaviside-Lorentz convention for α with $\rho_0 \simeq 1.19 \times 10^{-6}$ GeV/cm^3 [3]. The function $\delta(\nu)$ allows for a frequency-dependent time lag for emission from the GRB. To provide a context for our fit results, we first consider the determination of ε/M which would result were we to attribute the time lag associated with the radio afterglow of one GRB to a propagation effect. Choosing the GRB with the largest value of $K(z)/\nu^2$, we have a time lag of 3.31 ± 0.19 day associated with GRB 990510A at $z = 1.619 \pm 0.002$ measured at a frequency of $\nu = 4.8$ GHz [29]. Employing Eq. (8), setting $\delta = 0$, and noting that $K(z)/\nu^2 = 130 \pm 2\,\mathrm{Mpc\ GHz}^{-2}$ under the assumption that the error in z and in Ω_m with $\Omega_m + \Omega_\Lambda = 1$ are independent, the measured time lag fixes

$\varepsilon/M \simeq 1.8\times10^{-5}$ eV^{-1}. Turning to our data sample of 58 GRBs [29], we plot the measured time lag versus $K(z)/\nu^2$ in Fig. 1 and make a least-squares fit of Eq. (8) to determine $\tilde{A}_0$ and $\delta(\nu)$. We require $\tilde{A}_0 > 0$ as demanded by our model. The fit to the points with frequencies of $4.8-5$ GHz, with a scale factor in the uncertainty in $\tau/(1+z)$ of 130, to compensate for environmental effects in the vicinity of the emission from the GRB [29], yields $\chi^2/\mathrm{ndf} = 1.12$, with $\tilde{A}_0 = 0.0000 \pm 0.0049\,\mathrm{day\ GHz^2\,Mpc^{-1}}$ and $\delta = 1.9 \pm 0.4\,\mathrm{day}$. Thus $\tilde{A}_0 < 0.0096\,\mathrm{day\ GHz^2\,Mpc^{-1}}$ at 95% CL, and we determine

$$|\varepsilon|/M < 1.8\times10^{-5}\ \mathrm{eV}^{-1} \tag{9}$$

at 95% CL. The fits in the frequency window of $8-8.5$ GHz require a scale factor of 425 to yield $\chi^2/\mathrm{ndf} = 1.12$, with $\tilde{A}_0 = 0.012 \pm 0.017\,\mathrm{day\ GHz^2\,Mpc^{-1}}$ and $\delta = 0.15 \pm 0.71\,\mathrm{day}$. Thus $\tilde{A}_0 < 0.045\,\mathrm{day\ GHz^2\,Mpc^{-1}}$ at 95% CL, and our limit in this case is commensurately weaker.

The limit we find is the first direct observational limit which applies to dark matter in situ. We can compare our limit to those which arise from the nonobservation of the effects of millicharged particle production. For example, it is a factor of $2-3$ weaker than the strongest bound from laboratory experiments [25, 26], $|\varepsilon| < 3-4\times10^{-7}$ for $M \lesssim 0.05\,\mathrm{eV}$ [26]. The latter is also comparable to the model-independent bound arising from induced distortions in the cosmic microwave background radiation, though model-dependent constraints can be some two orders of magnitude stronger [27]. Limits also arise from stellar evolution constraints, for which the strongest is $|\varepsilon| < 2\times10^{-14}$ for $M < 5\,\mathrm{keV}$ [25]. In some models, the dynamics which gives rise to millicharged matter are not operative at stellar temperatures, so that those limits can be evaded [26].

We would like to estimate how much our limits would have to improve before the dispersive effects from ordinary charged matter would become apparent [30]. The only appreciable contribution to $\tilde{A}_0$ can come from free electrons. We estimate the cosmological free electron energy density ρ_e to be no larger than $\rho_e = (M_e/M_p)\rho_{\mathrm{cr}}\Omega_b \approx 0.130\,\mathrm{eV/cm^3}$ [3], where Ω_b is the fraction of the energy density in baryons with respect to the critical density today and M_e and M_p are the electron and proton mass, respectively. We replace ρ_0 with ρ_e and ε/M with $1/M_e$ in $\tilde{A}_0$ and find that the limit in Eq. (9) would have to improve by $\mathcal{O}(10^{-3})$ before the contribution from free electrons could be apparent.

Our limits on dark-matter properties rely on the validity of Eq. (1), which follows if $|n-1| \ll 1$. Although wave-like behavior is generally emergent for photon wavelengths which are large compared to the interparticle spacing, scattering in the forward direction is coherent irrespective of this criterion [20], and we expect Eq. (1) to persist for $\omega \gg N^{1/3}$ as well. Laboratory studies in dilute atomic systems do confirm the persistence of the index of refraction at low densities, though the parameters of these studies satisfy $\omega < N^{1/3}$ [31]. Any failure in Eq. (1) as $N \to 0$ has not been established.

In summary, we have described an observational program to deduce the properties of dark matter from the frequency-dependent propagation of light emitted from distant GRBs. Similar observational studies using high-energy photons have been suggested as a way of searching for Lorentz violation [18]. Our analysis relies on the theoretical constraints on the forward Compton amplitude emergent for $\omega \ll \omega_{\mathrm{th}}$; in this limit dispersive effects in the time delay depend on even powers of frequency only. As a first application of our method, we use observations of radio afterglows from GRBs at cosmological distances to find a limit on the millicharge of dark matter, which are within a factor of $2-3$ of the best laboratory limits from the nonobservation of millicharged particle production [26]. Note that our limit can be evaded by dilution. We set our limit from existing radio observations at roughly 5 GHz, so that our limit of $|\varepsilon|/M < 1.8 \times 10^{-5}$ eV^{-1} is operative if $\omega_{\rm th} \gg 3 \times 10^{-6}$ eV.

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